

Making the most of Unanticipated opportunities

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In a grade 9 (ages 14–15) introductory algebra lesson, the class is exploring how the m and b in the equation $y = mx + b$ are related to the graph of a linear function. During a class discussion about these relationships, a student asks if it is possible for the graph to have “two dots on the y -axis”—that is, whether it is possible for the linear function to have more than one y -intercept.

Before reading further, consider:

- How would you characterise this query? What is going on mathematically?
- What might be gained and what might be lost by pursuing this student idea at this moment?
- How might you as the teacher act on the student’s query?

Following calls for teaching to engage students in mathematics (e.g., NCTM, 2000), many authors have argued for the importance of focussing on student thinking and using it to build mathematical understanding (e.g., Breyfogle & Herbel-Eisenmann, 2004; Cavey & Mahavier, 2010; Foster, 2011; Stockero & Van Zoest, 2011). Smith, Hughes, Engle and Stein (2009) shared five practices for orchestrating classroom discourse around students’ thinking:

1. Anticipating;
2. Monitoring;
3. Selecting;
4. Sequencing; and
5. Connecting.

Their work emphasises the importance of teachers actively planning to elicit and make use of student input. Despite the best planning, though, not all student input can be anticipated or will fall within the teacher’s plan for the lesson. As in the vignette above, teachers are often faced with student input that interrupts the flow of a lesson and requires a decision about how to respond.

In the vignette, the student question introduces a topic—the definition of a function—that was not part of the plan for the lesson. The unanticipated question requires the teacher to decide whether the topic is worth departing from the plan to pursue at this time and, if so, how this might best be done. Within the complexity of classroom interactions, how does a teacher

recognise student input that is worth pursuing? If a student's contribution seems worth pursuing, how can it best be capitalised on? To begin to address these questions, we share research results that characterise unanticipated student mathematical input worth pursuing and teacher decisions in response to it that use students' ideas to enhance their understanding of mathematics.

Framing the work

A goal of our ongoing research is to better understand how student mathematical thinking that becomes public in a classroom can be used to support the learning of mathematics content and practices (e.g., Stockero & Van Zoest, 2011). Although there are often instances of student thinking that the teacher has intentionally cultivated to emerge at a particular time through a given task or a posed question, we were interested in learning more about instances that were not planned. We defined pivotal teaching moments (PTMs) as instances in a classroom lesson in which an interruption in the flow of the lesson provides the teacher an opportunity to modify their teaching in order to extend or change the nature of students' mathematical understanding. Here, we draw on our study of PTMs (as reported in Stockero & Van Zoest, 2013) to consider the potential of unanticipated student ideas that emerge during class discussions.

To better understand unanticipated student input, we examined over 45 hours of video of mathematics teaching in six teachers' classrooms. At the time of the data collection, the participating teachers were teaching mathematics in grades 8–12 (ages 13–18) in a variety of US school settings, including a rural school with a large Hispanic population, an urban school where approximately half the students were African-American, a suburban school with a predominately white population, and an alternative school for students who had not been successful in their local school. The topics taught included algebra, geometry, and trigonometry. Two of the teachers used investigative curricula, while the other four used transmission-based mathematics textbooks. Our analysis involved identifying PTMs, characterising PTMs in terms of type and their potential to support student learning, and examining teacher decisions and the resulting impact on student learning. We also investigated the relationships among these components.

Consistent with the mathematics education literature's description of high-quality learning opportunities (e.g., NCTM, 2000; Stein & Lane, 1996), PTMs were considered to have significant potential to support student learning if they involved rich mathematics or provided the opportunity to make connections among mathematical ideas. Often these PTMs provided a gateway to discussing important mathematical ideas that were not part of the planned lesson, but were an important part of the mathematical terrain the students were traversing. Moderate potential PTMs related to attributes of mathematics, such as its usefulness or coherency, or provided the opportunity to better understand procedures, definitions, or concepts.

In the following we draw on specific findings from our research to help teachers think about how they might make the most of unanticipated opportunities that occur in their classrooms. First we characterise high-leverage student input and then we discuss productive teacher responses. We define high-leverage student input as that which has significant potential for improving mathematics learning and productive teacher responses as those that use the student input to further student mathematics learning.

Characterising the unanticipated

The first step to capitalising on unanticipated opportunities is recognising them when they occur. Based on our work, we have identified five types of high-leverage student input: those that involve extending, sense-making, incorrect mathematics, mathematical contradiction, and mathematical confusion. Of the 39 PTMs identified in the data, 11 were characterised as extending, 13 as sense-making, 7 as incorrect mathematics, 2 as mathematical contradiction, and 6 as mathematical confusion. For more details about the potential of each PTM type, the teacher decisions made in response to the PTMs and the likely impact on student learning, see Stockero and Van Zoest (2013).

Extending

The opening vignette lesson focused on understanding how the m and b in the equation $y = mx + b$ relate to the graph of a linear function. By asking if it was possible to have more than one y -intercept, the student opened up the possibility of extending the lesson to make a connection with the definition of a function. In general, extending comments have significant potential to enhance learning because they provide opportunities to make connections between current learning and important mathematics from students' past or future learning.

Sense-making

Students' attempts to make sense of the mathematics in the lesson often provide opportunities to clarify or highlight critical mathematical ideas. For example, in one episode, a student who was trying to conceptually understand what was being presented as a procedure raised a question about why the procedure works. The student's push for meaning was an indicator to the teacher that the lesson was not making important connections and was a prompt to begin to do so. A variation of sense-making input occurs in lessons in which sense-making is the focus. In the process of trying to make sense of the mathematics, students sometimes over generalise or misconstrue aspects of the mathematics in unanticipated ways that, when surfaced, provide learning opportunities for the entire class. In our data set, unanticipated opportunities characterised as sense-making typically supported student learning by providing opportunities to better understand procedures, definitions, or concepts currently under discussion.

Incorrect mathematics

Unanticipated opportunities often occur when incorrect mathematical thinking or an incorrect solution is made public. Although some errors can be fairly inconsequential, as when based on an incorrect calculation or something else unlikely to interfere with students' mathematical understanding, other errors can affect what students take away from the lesson—either positively, if used to clarify an important mathematical idea, or negatively by introducing or reinforcing a misconception if not addressed. It is the errors that affect what students take away from the lesson that create high-leverage opportunities. For example, in one lesson, students were asked to construct a distance–time graph to model a situation in which a soccer ball was kicked into the air. One student's distance–time graph was a picture

of someone kicking a ball in the air and the path of the ball as it returned to the ground, rather than a graph relating the distance of the ball above the ground to the time that had lapsed since it was kicked. The student's mathematically incorrect representation of the situation provided an opportunity to clarify the difference between a mathematical representation of a situation and a picture of the situation—not the point of the lesson, but a common idea with which algebra students struggle (Wagner & Parker, 1993).

Mathematical contradiction

The occurrence of a mathematical contradiction almost always provides a high-leverage opportunity. This can be as straightforward as two different answers to a problem that clearly should have only one, or as complex as two competing interpretations of a mathematical situation. Regardless, the contradiction creates an opportunity for the teacher to bring to the students' attention the nature of mathematics that makes such contradictions unacceptable. It also provides an opportunity to highlight critical aspects of the mathematics at hand that can help students determine which of the options holds up under scrutiny. The very process of this scrutiny—the justification needed to support the different options and the making of a decision—can create a powerful learning opportunity.

Mathematical confusion

A student's expression of mathematical confusion can also provide an opportunity to enhance student understanding. It is important to distinguish general confusion—when students express that they do not know what is going on or that they cannot follow what someone has just explained—from mathematical confusion. Confusion seems to be a high-leverage opportunity when students can articulate mathematically what they are confused about. One example of this occurred when students were sharing their simplifications of expressions containing exponents (e.g., $(16x^2y^2)^{-1}(xy^2)^3$). As one student simplified an expression on the board, another student was able to point to the second step of the simplification as her point of confusion. This gave a mathematical focus to the confusion—the properties of exponents that were used in that step—and created an opportunity for the teacher to refocus the students on the meaning behind the procedures they were applying to the problem.

Responding to the unanticipated

Although teachers may choose to ignore or dismiss unanticipated student input, or briefly acknowledge the input without engaging with it, these types of inactive responses at best lead to a missed opportunity; at worst, students are left with incorrect or incomplete ideas about mathematics. In our study, inactive responses were associated with a positive impact only 6 % of the time. Furthermore, a teacher's failure to actively respond had a 29 % chance of being associated with a likely negative impact on student learning. For example, in the episode described in the vignette, we observed the teacher acknowledge, but dismiss the student's question by saying, "No, there cannot be two dots," and then praising the student for thinking hard. This response likely left the student (and possibly other students) with an incomplete understanding of why this cannot be the case.

Active responses, however, have the potential to positively impact student learning. Consistent with Walshaw and Anthony's (2008) conclusion from their synthesis of research on mathematics classroom discourse, our data suggests that a teachers' ability to notice and act on important mathematical opportunities that occur during instruction is critical to improving students' opportunities to learn mathematics. In our study, teachers' active response to such opportunities was associated with a positive likely impact on student learning 67% of the time. In your reflection on the teacher response to the vignette, you may have decided to emphasise mathematical meaning, find out more about what the student was thinking, or go beyond the topic the students were working on in the lesson to revisit and make connections to past learning. These were some of the types of active responses we identified in our data.

To illustrate, first consider an alternate teacher response to the student's input in the vignette—using the students' question as an opportunity to revisit the definition of function. This would be an example of the teacher deciding to go beyond the planned lesson to make a connection between the definition of function that had been introduced in an earlier lesson and the graphs of the linear functions that were being studied in the current lesson. This decision would provide an important opportunity to revisit this foundational idea in the study of algebra and deepen students' understanding of what makes a graph a function.

In another lesson we observed, students had been working in small groups to create a graphical model of a rabbit jumping over a fence, given the parameters of a three-foot-high fence and the rabbit leaving and returning to the ground 4 feet from either side of the fence. During the ensuing whole-class discussion, the students were engaged with the context and figuring out what was going on mathematically. The teacher used the student input, "All the values only gonna be on our line, though, right?", as an opportunity to engage the class in considering points not on the curve and, in doing so, highlighted what the points on a graph represent. The unanticipated student input provided the opportunity to emphasise an important mathematical idea—a graph represents the set of ordered pairs that satisfy a particular rule—and enhance students' mathematical understanding.

It is important to note that active decisions do not guarantee enhanced student learning; our work suggests that implementation matters. There were times in our data when teachers attempted to actively respond to students' comments, but because they misread the mathematics underlying the comments, opportunities to enhance learning were missed. However, when active responses are effectively implemented, they help students to make sense of mathematical ideas, clarify their thinking, and make connections among mathematical ideas.

Conclusion

Many teachers are establishing classroom environments where students are encouraged to ask questions and make observations about the mathematics they are learning. Having students share their thinking, however, creates a new problem: how to productively use the thinking that emerges, particularly that which is unanticipated. Being aware of the five types of potential high-leverage student input found in our research—extending, sense-making, incorrect mathematics, mathematical contradiction, and mathematical confusion—has the potential to help teachers avoid

the phenomenon of failing to focus attention on unexpected events, what Simons (2000) called *inattentional blindness*. That is, knowing that these types of input often represent high-leverage thinking is an important first step to recognising and acting on them in a way that develops students' mathematical understandings.

We have found that high-leverage instances of student thinking are most likely to occur when students are actively engaged in the mathematics lesson—regardless of the nature of the classroom teaching or the curriculum used. They can occur in classrooms where students are doing mathematics themselves and sharing their thinking with their classmates, as well as in classrooms where students are listening to the teacher present information and asking questions about what they are hearing. This points to the importance of all teachers improving their ability to notice and productively respond to unanticipated student input in their classroom teaching. Given the myriad of moment-by-moment decisions teachers are faced with and limited teaching time, recognising high-leverages instances of student thinking is critical to making informed decisions about which student thinking might be productive to pursue.

We challenge you to make the most of unanticipated opportunities in your teaching. Rather than ignoring or dismissing student input that is unexpected, consider whether the input reflects one of the types of high-leverage student thinking identified in this paper. If so, consider how best to actively respond. Do you need to find out more about the mathematical thinking behind the input? Does the input provide an opening to extend the mathematics or make connections among mathematical ideas? Can it be used to emphasise the meaning of the mathematics under consideration? These are all starting points for using unanticipated student input in ways that support the development of students' mathematical understanding.

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